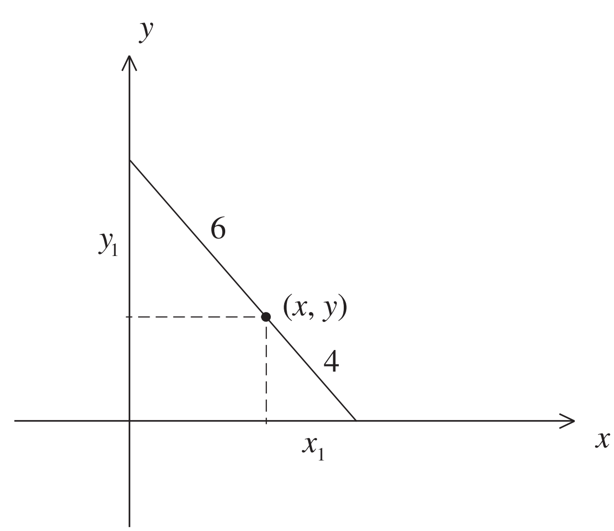
Exam D (Part I) **Name**

1. The graph of the equation $x^2 + 6x + y^2 - 4y - 36$ is a circle. Find the center and the radius of this circle (by completing squares).

2. Let θ be any angle. Place the unit circle on an xy-coordinate system and explain why the formulas $\sin(\pi - \theta) = \sin\theta$ and $\cos(\pi - \theta) = -\cos\theta$ hold.

3. A 10 foot long ladder leaning against a vertical wall is sliding on the slippery horizontal surface that supports it. The figure illustrates the ladder in typical position within an xy-coordinate system. The point $P = (x, y)$ is fixed on the ladder 6 feet from its upper tip (and hence 4 feet from its lower tip). Show that as the ladder slips, the point P describes an ellipse with semimajor axis 6 and semiminor axis 4. (Start by using similar triangles to determine the lengths x_1 and y_1 in terms of the coordinates x and y respectively.)



4. Use Leibniz's tangent method to compute the slope of the tangent to the curve $y^2 = x^2 + 2$ at any point $P = (x, y)$. Make use of your answer to compute the derivative of the function $f(x) = \sqrt{x^2 + 2}$.

5. Consider the graph of the function $f(x) = x^2$ over the interval $0 \leq x \leq 2$ and sketch it. Place a rectangle under this graph so that its bottom side lies on the x-axis between 0 and 2.

i. Determine the largest area that such a rectangle can have. What are the dimensions of the rectangle with the largest area?

ii. What fraction of the total area under the graph of $f(x) = x^2$ over $0 \leq x \leq 2$ does this rectangle take up?

6. Consider the function $g(x) = 9 - x^2$ with $0 \leq x \leq 1$. Insert the points $0 \leq 0.3 \leq 0.5 \leq 0.8 \leq 1$ on the x-axis between 0 and 1 and compute the sum $g(x) \cdot dx$ that this set of points determines. Do so with three decimal place accuracy. This sum is an approximation of the area under the graph of $g(x) = 9 - x^2$ over $0 \leq x \leq 1$. Sketch what is going on in the space below. Use the fundamental theorem of calculus to compute this area precisely.

7. Consider the graph of $f(x) = x^3$ for $0 \leq x \leq 3$. Find the volume of the solid obtained by rotating the region under the graph (and above the x-axis) one revolution around the x-axis.

8. Use the powerseries $\frac{1}{1+x} \approx 1 - x + x^2 - x^3 + x^4 - x^5 + \ldots$ to approximate $\displaystyle\int_0^1 \frac{1}{1 + \frac{1}{8}x^3}\, dx$ with four decimal accuracy.

9. A point is moving in the x-y plane starting at time $t = 0$. Its x and y coordinates at any time $t > 0$ are given by $x(t) = t^{\frac{1}{2}}$ and $y(t) = t + 2$. Determine an equation of curve in the x-y plane along which the point travels and sketch its graph below.

i. Compute the velocities in the x- and y-directions and the velocity overall.

ii. Compute the accelerations in the x- and y-directions.

iii. By discussing the velocities and forces (use $F = ma$ with $m = 1$) in the x- and y-directions, describe how the point traces out its path.